ARTICLE NO. 79-4138

Required Radar Ranges for AEW Aircraft

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In designing an airborne early warning system to warn of incoming air raids, one should match the radar range to the velocity of the incoming raid, the reaction time of one's interceptors, and the capacity of the airborne early warning system to flee to safety. This paper, through the application of simple analytical models, investigates the minimum radar ranges necessary to permit interdiction of the raid as well as escape to safety of the airborne early warning system. These radar ranges are dependent on characteristics of all the aircraft in the scenario. The results of many calculations are presented, showing the sensitivity of radar range to selected parameters.

Introduction

AIRBORNE early warning (AEW) aircraft, such as AWACS, provide an effective means of securing advance notification of an enemy air raid. However, to optimally design such a system, one must contend with a number of issues. An aircraft possessing great speed probably cannot carry a very powerful radar, while an aircraft with good payload and endurance characteristics requires a radar of sufficient range to permit it time to escape an airborne threat against it. Also bearing on the problem are the speeds of the incoming raid as well as the reaction time of one's own interceptors. Of course, sufficient combat air patrol (CAP) aircraft can protect the AEW aircraft, but provision of CAP is extremely expensive. Thus, the topic considered here is important from a force structure cost point of view.

This paper presents a simple model with which to examine the following question: What is the minimum range required for airborne early warning radar aircraft such that 1) they can provide sufficient early warning of a bomber raid so that interceptors can interdict the raid, and 2) they have sufficient time to flee to safety if attacked?

The radar ranges determined by this model will be the minimums required whether the radars are operating in a clear or jamming environment, but to achieve these ranges against a specified jamming threat is a complex design question beyond the scope of this article.

The context in which we will place this question is depicted in Fig. 1, which shows two airborne radar aircraft providing warning in a barrier using interceptor aircraft from bases A and B. More airborne radar aircraft could be in the barrier to the left and right of the two aircraft, but these need not be considered in the analysis since they would be located closer to the airbases. We do not treat cases with more than two aircraft between airbases, although this approach can easily be extended to such cases.

We denote by C the forward point of intersection of the two radar coverage disks, and position the aircraft symmetrically on the lines $\overline{AC}$ and $\overline{BC}$. We assume that the bomber raid cannot circumvent the barrier by going around the radar coverage, and that to provide the least warning the raid enters coverage at point C and proceeds along a path perpendicular to the line $\overline{AB}$.

In addition to the main bomber raid, the enemy may send chasers after the airborne radar in an attempt to destroy it. The chasers are assumed to travel at a velocity greater than or equal to that of the bombers. They may break off the main bomber raid at point C, or they may break off earlier and approach the radars from a different angle. We will assume, however, that the chasers and the bombers enter radar coverage simultaneously. There are two reasons that this assumption is acceptable:

1) if the bombers enter first, warning of the raid is obtained, while, relatively, the chaser threat is delayed, providing the radars with a less demanding escape requirement; and

2) if the chasers enter first, the interceptors will have increased warning time against the bomber raid, facilitating interdiction of the bombers.

Received May 23, 1978; revision received Jan. 29, 1979.

Index category: Military Missions.

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Early Warning and Intercept

The first mission of the airborne radars is to provide sufficient warning of the raid so that interceptors launched from A and B can interdict the raid. We define this to mean that the first wave of interceptors will arrive at point D, shown in Fig. 2, at the same time as the first wave of bombers arrives at point D.

More explicitly, the bombers travel from point C toward D at speed v_b, while the interceptors, after a time delay to t_i (to account for degree of alertness of the interceptors), proceed from the bases toward point D with speed v_i. If all these elements are to meet at D we have the following relationship:

$$\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2 = |\overline{AC}|^2 = |\overline{BC}|^2 \qquad (1)$$

where $|\overline{AB}|$ denotes the length of the line $\overline{AB}$ (and so forth).

By symmetry, we need look only at the ACD triangle, and if we denote by E the location of the radar on AC, then by observing that $|\overline{CE}| = R$, the radar range, we can rewrite Eq. (1) as

$$\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2 = \left(R + |\overline{AE}|\right)^2 \qquad (2)$$

Equation (2) must hold if early warning, as we have specified it, is to be adequate.

Escape to Line Between A and B

We now superpose on the early warning mission the requirement that the radar range must be sufficient to permit the airborne radar to escape to safety. Two types of escape will be examined. In the first, the airborne radars flee directly away from the chasers and are considered safe when they cross the line through bases A and B, where, it is presumed,

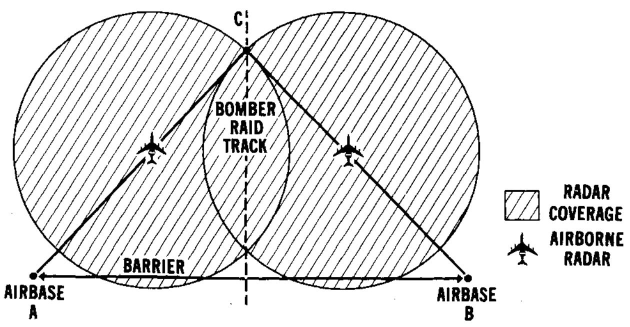

Fig. 1 Airborne radar barrier configuration.

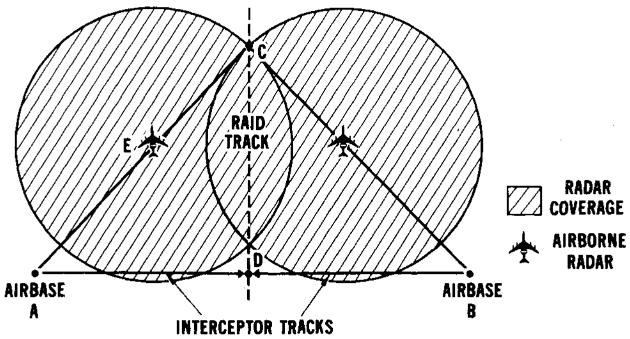

Fig. 2 Interdiction of the raid.

interceptors attacking the main raid will be available to attack the chasers. This situation is shown in Fig. 3, for half the barrier (assuming that the other half is a mirror-image).

The chasers enter radar coverage at point F, at the same time as the raid enters at point C. The airborne radar originally located at point E flees toward point H, with the chasers in pursuit, along line $\overline{FH}$, which forms an angle θ with the line $\overline{AC}$. The airborne radars require time t_r to decide whether to flee and then proceed at velocity v_r. The chasers fly at speed v_c. If the radars can reach point H before the chasers are within lethal range l_c, then the radars are considered safe.

From Fig. 3 and Eq. (1) it is evident that

$$\tan\alpha = \frac{|\overline{AB}|/2}{v_c\left(t_i + \dfrac{|\overline{AB}|}{2v_i}\right)} \tag{3}$$

Also,

$$\beta = \alpha - \theta$$

so that

$$|\overline{EG}| = |\overline{AE}|\cos\alpha$$

Looking at triangle EGH, observe that

$$\cos(\alpha-\theta) = |\overline{EG}|/|\overline{EH}| = (|\overline{AE}|\cos\alpha)/|\overline{EH}| \tag{4}$$

The time required for the radar to reach point H is

$$(|\overline{EH}|/v_r) + t_r$$

Assume that, in this time, the chasers have arrived within l_c of the radars, so that they have flown $R + |\overline{EH}| - l_c$. Equating times of flight we have

$$(|\overline{EH}|/v_r) + t_r = (|\overline{EH}| + R - l_c)/v_c \tag{5}$$

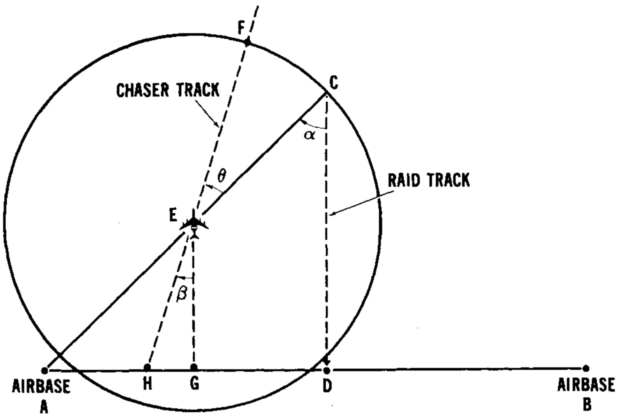

Fig. 3 Radar escapes to the line through A and B.

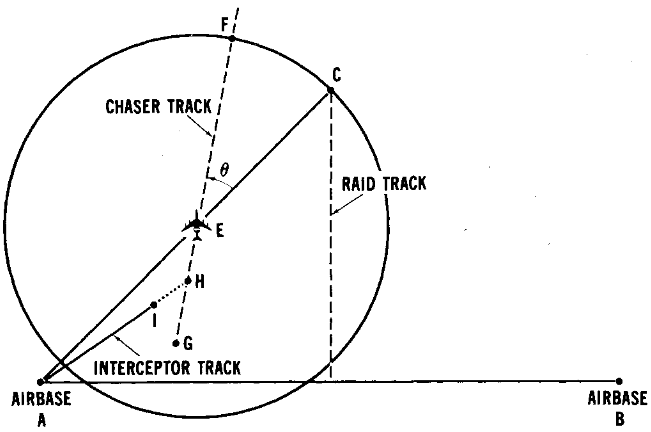

Fig. 4 Radar escapes to interceptors flying out from base.

Rearranging Eq. (5) yields

$$|\overline{EH}| = \frac{v_r}{(v_c - v_r)}\{R - l_c - t_r v_c\} \tag{6}$$

Combining Eq. (6) with Eq. (4) we have

$$|\overline{AE}| = \frac{\cos(\alpha-\theta)}{\cos\alpha}\cdot\frac{v_r}{v_c - v_r}\{R - l_c - t_r v_c\}$$

and substituting into Eq. (2) we obtain

$$R = \left(1 + \frac{\cos(\alpha-\theta)}{\cos\alpha}\cdot\frac{v_r}{v_c - v_r}\right)^{-1}\left[\frac{v_r}{v_c - v_r}\cdot\frac{\cos(\alpha-\theta)}{\cos\alpha}\right.$$
$$\left.\times\{l_c + t_r v_c\} + \sqrt{\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2}\right] \tag{7}$$

Since we know $|\overline{AB}|$ and since α can be obtained from Eq. (3), R can be computed from Eq. (7).

Escape to Interceptors Flying Toward Fleeing Airborne Radars

In the second type of radar escape, the airborne radars flee directly away from the chasers as before. Now, however, instead of being saved by crossing the baseline $\overline{AB}$, radars are saved by interceptors, which must arrive within interceptor lethal range l_i of the chasers when (or before) the chasers are within chaser lethal range l_c of the radars.

This is illustrated in Fig. 4. As before, the chasers enter radar coverage at point F and pursue the radars along line $\overline{FG}$. The radars wait a time t_r before fleeing the chasers. After a

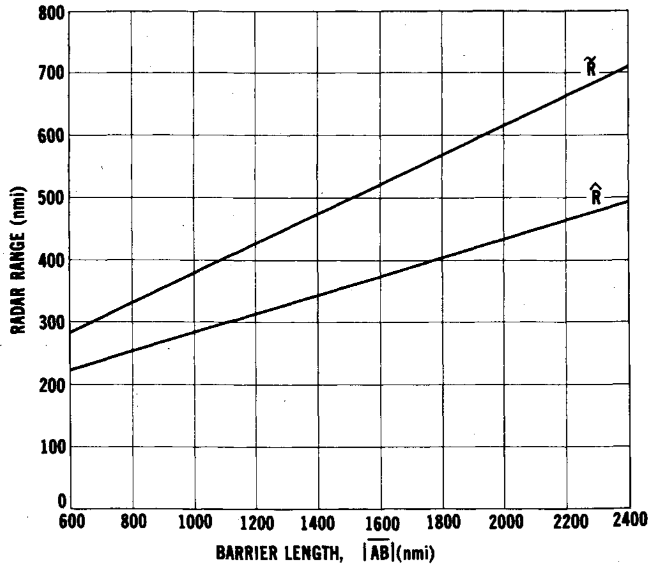

Fig. 5 Required radar range as a function of barrier length.

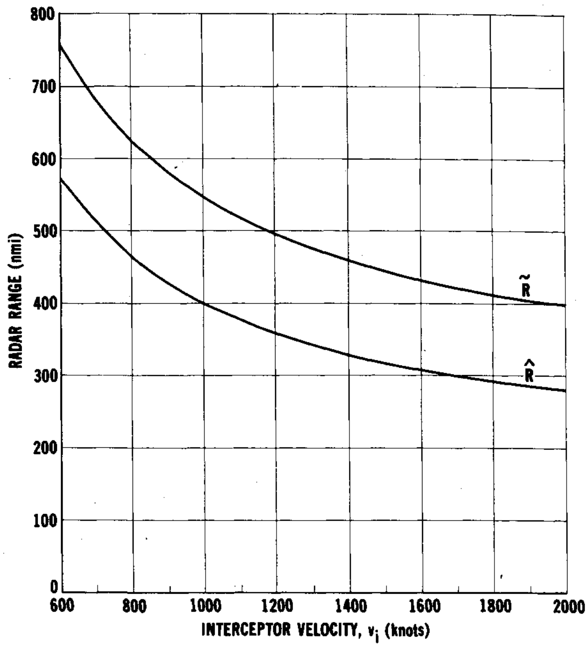

Fig. 7 Required radar range as a function of interceptor velocity.

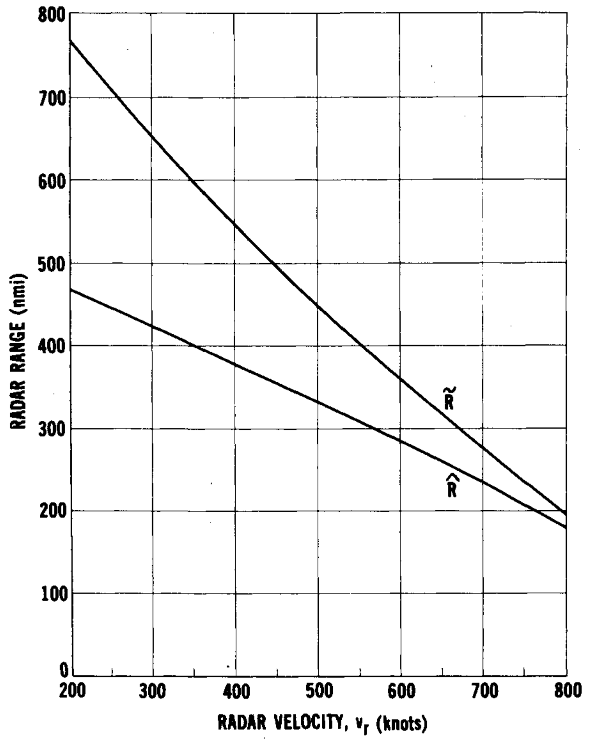

Fig. 6 Required radar range as a function of radar velocity.

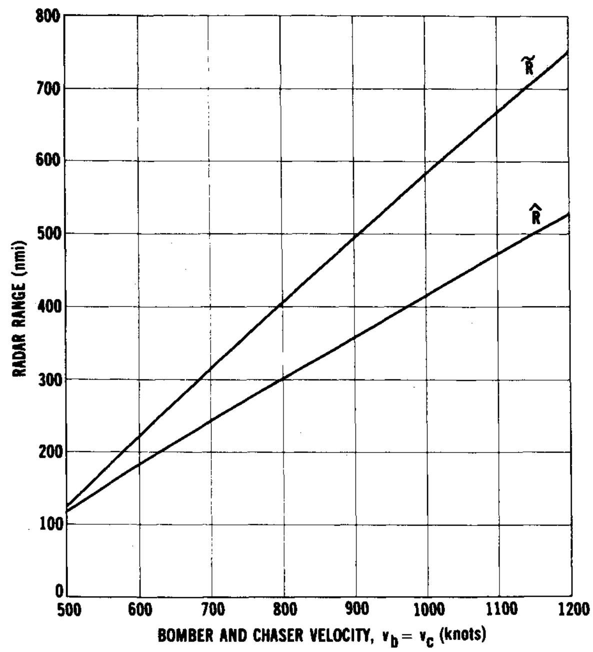

Fig. 8 Required radar range as a function of bomber and chaser velocities.

delay t_i, the interceptors fly along line $\overline{AI}$ in order to be at point I when the chasers are at H and the radars are at G, where $|\overline{HI}| = l_i$ and $|\overline{GH}| = l_c$.

The radars require $|\overline{EG}|/v_r + t_r$ to arrive at point G. If the chasers are to be at point H, we have

$$|\overline{EG}|/v_r + t_r = (R + |\overline{EG}| - l_c)/v_c \tag{8}$$

which yields

$$|\overline{EG}| = \frac{v_r}{v_c - v_r}(R - l_c - t_r v_c) \tag{9}$$

In time $(|\overline{EG}|/v_r) + t_r$ the interceptors have flown

$$v_i[(|\overline{EG}|/v_r) + t_r - t_i] = |\overline{AF}| \tag{10}$$

Substituting the right-hand side of Eq. (9) for $|\overline{EG}|$ in Eq. (10) we have

$$|\overline{AI}| = \frac{v_i}{v_c - v_r}\cdot R - \frac{v_i}{v_c - v_r}(l_c + t_r v_c) + v_i(t_r - t_i) \tag{11}$$

Now, since $|\overline{AH}| = |\overline{AI}| + l_i$ we get

$$|\overline{AH}| = \frac{v_i}{v_c - v_i}\cdot R - \frac{v_i}{v_c - v_r}(l_c + t_r v_c) + v_i(t_r - t_i) + l_i \tag{12}$$

By the law of cosines we know that

$$|\overline{AH}|^2 = |\overline{AE}|^2 + |\overline{EH}|^2 - 2|\overline{AE}|\cdot|\overline{EH}|\cos\theta \tag{13}$$

Now, $|\overline{AH}|$ is given by Eq. (12) and $|\overline{EH}|$ equals $|\overline{EG}| - l_c$ where $|\overline{EG}|$ is given by Eq. (9). Further, from Eq. (2) we know that

$$|\overline{AE}| = \sqrt{\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2} - R$$

Substituting all this into Eq. (13) yields

$$\left[\frac{v_i}{v_c - v_r}\cdot R - \frac{v_i}{v_c - v_r}(l_c + t_r v_c) + v_i(t_r - t_i) + l_i\right]^2 = \left\{\sqrt{\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2} - R\right\}^2 + \left\{\frac{v_r}{v_c - v_r}(R - l_c - t_r v_c) - l_c\right\}^2$$
$$-2\left\{\sqrt{\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2} - R\right\}\left\{\frac{v_r}{v_c - v_r}(R - l_c - t_r v_c - l_c\right\}\cos\theta \qquad (14)$$

which can be expanded and rewritten as

$$0 = R^2\left[1 - \left(\frac{v_i}{v_c - v_r}\right)^2 + \left(\frac{v_r}{v_c - v_r}\right)^2 + 2\left(\frac{v_r}{v_c - v_r}\right)\cos\theta\right] + R\left[2\left(\frac{v_i}{v_c - v_r}\right)\left\{\frac{v_i}{v_c - v_r}\cdot(l_c + t_r v_c) - v_i(t_r - t_i) - l_i\right\}\right.$$
$$-2\sqrt{\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2} - 2\cdot\frac{v_r}{v_c - v_r}\cdot\left\{\frac{v_r}{v_c - v_r}\cdot\left(l_c + t_r v_c\right) + l_c\right\} - 2\left\{\frac{v_r}{v_c - v_r}\cdot\sqrt{\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2}\right.$$
$$\left.\left.+\frac{v_r}{v_c - v_r}(l_c + t_r v_c) + l_c\right\}\cos\theta\right] + \left[\left(\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_I}\right)^2\right) - \left\{\frac{v_i}{v_c - v_r}(l_c + t_r v_c) - v_i(t_r - t_i) - l_i\right\}^2\right.$$
$$\left.+\left\{\frac{v_r}{v_c - v_r}(l_c + t_d v_c) + l_c\right\}^2 + 2\sqrt{\left(\frac{|\overline{AB}|}{2}\right)^2 + v_b^2\left(t_i + \frac{|\overline{AB}|}{2v_i}\right)^2}\cdot\left\{\frac{v_r}{v_c - v_r}(l_c + t_r v_c) - l_c\right\}\cos\theta\right] \qquad (15)$$

Finally, R can be obtained from Eq. (15) using the quadratic formula.

Results

The models given above are sufficiently simple that they can be programmed onto some hand calculators, yet they are sensitive to many of the major parameters of the airborne radar coverage problem. In what follows, we present some parametric excursions to illustrate the behaviors of the models as each of the parameters is varied.

The required radar range to provide early warning and escape to a line between A and B is denoted by $\tilde{R}$ and computed by Eq. (7). The required radar range to provide early warning and escape to interceptors flying from the base is denoted by $\hat{R}$ and computed from Eq. (15).

Because considerations such as altitude, target size and engineering constraints have not been included in this investigation, some of the radar ranges reported on the following pages exceed line of sight distances or are unlikely from a technical point of view. The reader can compute for himself line of sight distance D (in n.mi.) using the following approximation:

$$D = 1.2(\sqrt{A_r} + \sqrt{A_t}) \qquad (16)$$

where A_r is the radar altitude (in feet) and A_t is the threat altitude (in feet). D will therefore vary from about 200 n.mi. for a 20,000-ft radar against an "on the deck" 500-ft threat to over 600 n.mi. for an 80,000-ft radar against a high-altitude 50,000-ft threat.

A base case is chosen, from which variations are performed. In these excursions, only the parameter under investigation is varied; the rest are held constant at base-case values. The base-case parameter values are as follows:

$\lvert\overline{AB}\rvert$	= 1500 n.mi.
v_b	= 900 n.mi./h
v_c	= 900 n.mi./h
v_r	= 450 n.mi./h
v_i	= 1200 n.mi./h
t_r	= 5 min
t_i	= 10 min
l_c	= 50 n.mi.
l_i	= 50 n.mi.
θ	= α (see Fig. 3)

θ is chosen so that the chaser tracks are parallel to the bomber tracks. The variations made from this base case are discussed in the following.

Series 1: Required Radar Range as a Function of Barrier Length

Figure 5 depicts the relationship between the required radar ranges $\tilde{R}$ and $\hat{R}$, and barrier length $|\overline{AB}|$. As one would expect, $\hat{R}$, the radar range required if the airborne radars are to be saved by interceptors, is uniformly less than $\tilde{R}$, the range required if the radars must return to baseline $\overline{AB}$ to be saved. Moreover, $\hat{R}$ grows more slowly with increasing barrier length, growing by only 0.15 n.mi. per additional nautical mile of barrier length as opposed to the 0.24 n.mi. per additional nautical mile of barrier length required by $\tilde{R}$.

Series 2: Required Radar Range as a Function of Radar Velocity

As radar velocity is increased, the radar range required decreases, as shown in Fig. 6. Although $\tilde{R}$ decreases faster with respect to increasing radar velocity, $\hat{R}$ is always less.

Series 3: Required Radar Range as a Function of Interceptor Velocity

If interceptor velocity is increased, then less warning is needed to interdict the bomber raid and therefore radar range can be decreased. It also happens that the airborne radars can be stationed closer to their bases, facilitating their escape from the chasers by either of the two modes we are considering. Figure 7 shows the effect on $\tilde{R}$ and $\hat{R}$ of various interceptor velocities. The two curves are not quite parallel: $\hat{R}$ is 179 n.mi. less than $\tilde{R}$ when v_i is 600 n.mi./h, but this ad-

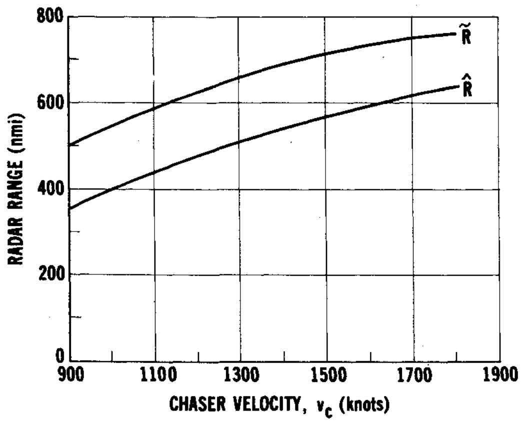

Fig. 9 Required radar range as a function of chaser velocity.

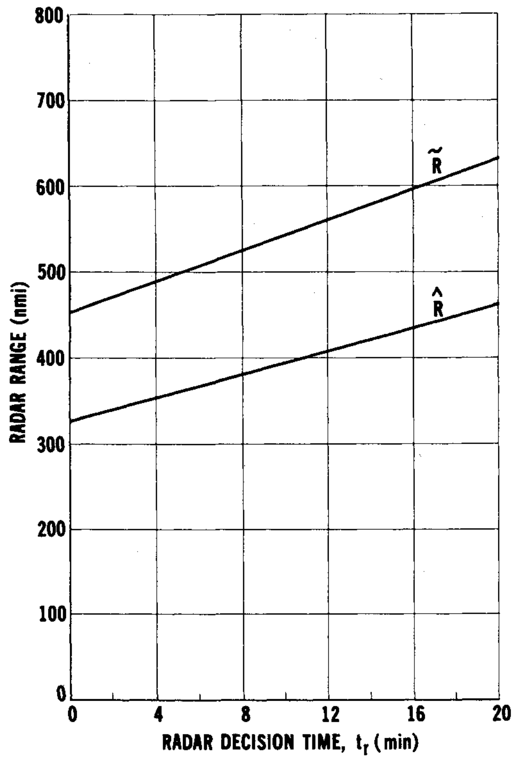

Fig. 10 Required radar range as a function of radar decision time.

vantage drops to only 117 n.mi./h when interceptor velocity is increased to 2000 n.mi./h.

Series 4: Required Radar Range as a Function of Bomber and Chaser Velocity

For this series, bomber velocity is increased. Because we assume that chaser velocity is not less than bomber velocity, chaser velocity is always equal to bomber velocity. As v_b increases, more warning is necessary if the raid is to be interdicted. Figure 8 shows that while $\tilde{R}$ and $\hat{R}$ differ little for low velocity raiders, $\tilde{R}$ increases much more quickly than $\hat{R}$ as v_b increases. Although the curves appear to be linear, they are not; rather, they are slightly concave.

Series 5: Required Radar Range as a Function of Chaser Velocity

The previous series treats the case in which bomber and chaser velocities are increased. This series, illustrated in Fig. 9, examines the effect on radar range as chaser velocity only is increased. As might be predicted, the requirements on $\tilde{R}$ and $\hat{R}$ are less stringent than for the cases in which both bomber and chaser velocities are increased.

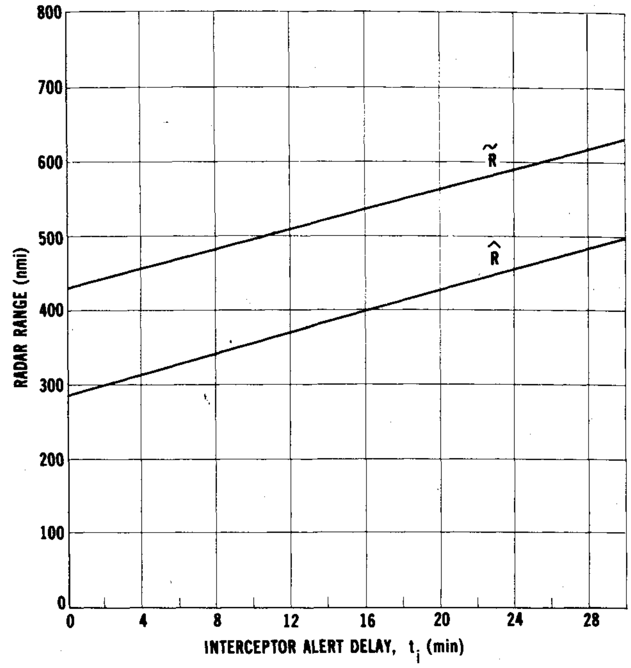

Fig. 11 Required radar range as a function of interceptor alert delay.

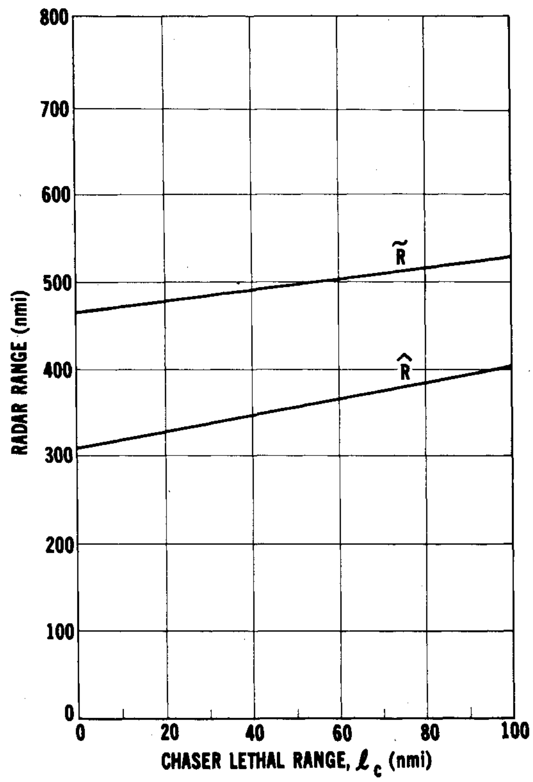

Fig. 12 Required radar range as a function of chaser lethal range.

Series 6: Required Radar Range as a Function of Radar Decision Time

Radar decision time is the time the radar remains on station before evading the chasers. It does not affect, in any way, the raid interdiction mission of the interceptors. Although it is evident from Eq. (7) that $\tilde{R}$ is linear in t_r, it is not clear that $\hat{R}$ is also linear in t_r. From Fig. 10, however, $\hat{R}$ appears to be, for all practical purposes, a linear function of t_r. The value of $(\tilde{R}-\hat{R})$ ranges from a minimum of 120 n.mi. when $t_r=0$ to a maximum of 171 n.mi. when $t_r=20$ min.

Therefore, as t_r increases, the advantage of having interceptors save the airborne radars increases.

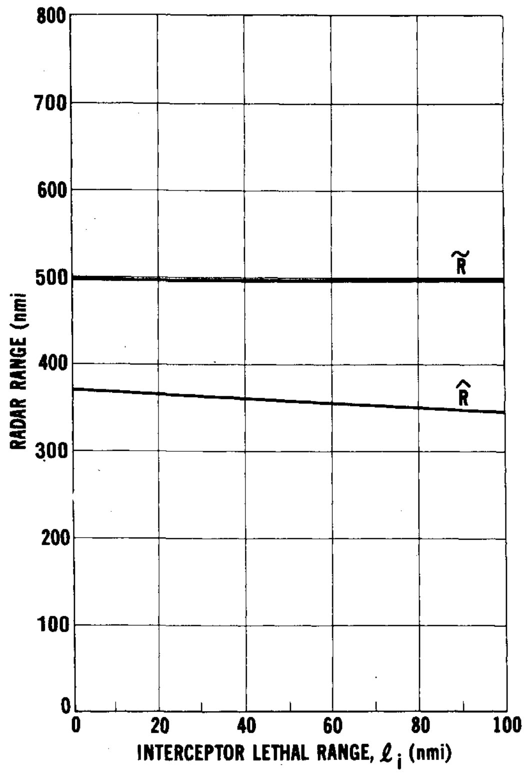

Fig. 13 Required radar range as a function of interceptor lethal range.

Series 7: Required Radar Range as a Function of Interceptor Alert Delay

As t_i, the time delay between warning and interceptor launch, increases, the more advance notice of the impending raid is necessary if it is to be interdicted. Figure 11 shows how $\tilde{R}$ and $\hat{R}$ vary with t_i. $\hat{R}$ is slightly more sensitive to changes in t_i, with $\tilde{R}$ increasing by 6.7 n.mi. for each minute increase in t_i as compared with $\hat{R}$ increasing by 6.9 n.mi. for each minute increase in t_i.

Series 8: Required Radar Range as a Function of Chaser Lethal Range

Figure 12 indicates that $\hat{R}$ is more sensitive to increases in chaser lethal range. Nevertheless, even $\hat{R}$ is at least 125 n.mi. less than $\tilde{R}$ for values of l_c up to 100 n.mi.

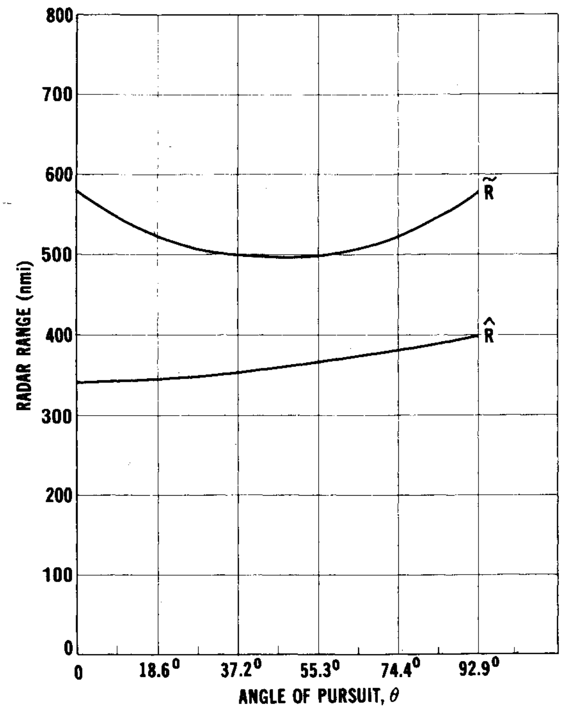

Fig. 14 Required radar range as a function of chaser angle of pursuit.

Series 9: Required Radar Range as a Function of Interceptor Lethal Range

Because interceptor lethal range l_i is not explicitly considered in the raid interdiction mission or in the escape of radars that flee to the baseline, it is clear that varying l_i can have no effect on $\tilde{R}$. Surprisingly, however, varying l_i has only a slight effect on $\hat{R}$. This is shown in Fig. 13. Over the range $l_i = 0$ to $l_i = 100$ n.mi., $\hat{R}$ decreases by only 24 n.mi.

Series 10: Required Radar Range as a Function of Chaser Angle of Pursuit

As noted above, we assume in the base case that the chasers pursue the radar along a track parallel to the main raid; hence the pursuit angle $\theta = \alpha = 46.5$ deg (in this case). Figure 14 shows that this value of θ actually is at a minimum of $\tilde{R}$. This is because along this track lies the shortest distance to the baseline. $\hat{R}$, on the other hand, is monotone increasing in θ, for the greater the value of θ, the farther the interceptors will have to fly to protect the radar.